# Multilevel Halftoning using Bilevel Quantizers

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# Abstract

The introduction of digital printing devices, capable of printing multiple shades of gray, has triggered research focusing on multilevel halftoning algorithms. A major problem associated with conventional multilevel halftoning is the introduction of unwanted texture near the intermediate gray levels, in the printed image. In this paper, a novel halftoning algorithm, based on gray level separation, has been introduced and tested on gray scale ramps, using trilevel halftoning. With this method, we can control the proportion of gray and black pixels to represent a particular shade of gray. For certain combinations of the gray and black distribution curves, elimination of the undesirable banding artifacts near the intermediate gray level, in the output, has been achieved.

#### 1. Introduction

Over the years, there has been significant technological improvements in printing devices, in an effort to achieve superior image quality. Two major improvements in inkjet printers have been the increase in the resolution of the printer head and the use of multiple inks to represent intermediate tones. However, since there are technological constraints upon the increase in the resolution of the printing head beyond a certain limit (1440 dpi), any further improvement in image quality can only be achieved by employing more shades of gray. With the availability of such printing devices, capable of producing multiple shades of gray, a lot of research today focuses on developing appropriate multilevel halftoning algorithms for these devices.

Conventional error diffusion, for binary devices, can easily be extended to the multilevel case, by replacing the thresholding stage with a multilevel quantizer [1]. However, the images halftoned using this conventional multilevel halftoning algorithm exhibit banding artifacts in the regions of intermediate printable gray levels, the reason being that at the intermediate gray level there is no quantization error as we have the ink for that tonal level available. In the neighborhood of this zero error region, there is a sparse distribution of the black and white pixels, in an



Figure 1: Halftoned gray scale ramp (a) binary (b) trilevel. Banding can clearly be seen in the ramp generated using trilevel error diffusion around gray level 0.5.

otherwise constant gray region. This appears as a banding artifact to the human observer. The effect can clearly be observed in Fig. 1, showing a halftoned gray scale ramp, using bilevel and trilevel error diffusion.

A number of algorithms have been proposed to eliminate the above problems with multitones. All these schemes focus on equalizing the distribution of mean square error, between the halftoned and the original image, distributing the MSE, uniformly, over all the gray levels. Miller and Smith proposed a LUT based multitoning scheme [2], allowing the growth of any conceivable dot growth pattern. Yu et al. proposed an over-modulation scheme [3] to overcome the same problem. These two schemes are based on multilevel stochastic screens. Several optimizations to the conventional multilevel error diffusion algorithm were also proposed to solve the texture problem in multitones. Ochi [4] proposed an algorithm to reduce contouring in multilevel error diffusion, iterating the error diffusion with a layered structure. More recently, Sugiura and Makita [5] have suggested a new multilevel error diffusion method, addressing the same contouring problem.

In this paper, we introduce a novel multitoning technique, based on conventional binary error diffusion algorithm, addressing the problem of banding in multitones. The proposed method is based on the decomposition of the original image into the intermediate printable gray scale images, using some constrained modulation function, halftoning each channel using bitonal error diffusion algorithm, in a correlated fashion, and recombining the halftoned channels to get the result.

# 2. Multitoning using Bilevel Quantizers

The new multitoning technique, introduced in this section, is based on the concept of gray level separation. The idea behind gray level separation is that, a continous tone image can be decomposed into the constituent gray scale images, of printable gray levels, according to some constrained modulation function. Each image is halftoned, independently, using a bilevel quantizer, as against the conventional schemes employing the multilevel quantizer for multitoning. The independently halftoned images are, then, recombined to give the final halftoned image.

We can represent a gray level of a continuous tone image by a linear combination of a finite number of intermediate (printable) gray levels. In multilevel halftoning, black and gray pixels along with the background pixels (white), in appropriate proportions, give the perception of a shade of gray. However, it is not well defined, in what proportion the gray, black and white pixels must be present to represent a particular gray level of the input image.

In case of trilevel halftoning, in addition to black, we have one more intermediate printable gray level. Hence the continuous tone (intensity) image can be represented by a linear combination of the black and 50% gray levels. This linear relationship between the continuous tone image and the printable gray levels can be expressed as:

$$xB = bB + gG, \quad 0 \le x \le 1 \tag{1}$$

where x is the percentage gray level of the intensity image, b and g are the proportions of the black and gray levels, composing the input image. In fact, b and g are the factors controlling the number of pixels of the black and gray inks, appearing in the output (halftoned) image, used to represent a particular shade of gray. B and G are the intensities of the black and gray inks. Assuming ideal inks, the relationship between the two ink intensities, for trilevel halftoning is G = B/2. Thus we have from (1).

$$x = b + \frac{g}{2}, \quad 0 \le x \le 1$$
 (2)

In general for a M-ary printing device, we have M - 2intermediate gray levels, in addition to the black level, and we have a more general relationship between the input



Figure 2: Gray level transform curves for two conceivable patterns. (a) shows a symmetrical distribution of gray ink around gray level 0.5 and (b) shows an asymmetrical distribution of gray ink.

image intensity and the constituent ink intensities, in the halftoned output. This relationship is given by the equation:

$$x = b + \sum_{i=1}^{M-2} a_i g_i, \quad 0 \le x \le 1$$
 (3)

where  $a_i$  is the proportion of ink of gray level  $g_i$ . A unique assignment for the  $a'_i s$  does not exist in (3). In fact, an infinite number of assignments of the  $a'_i s$  are possible.

Without loss of generality, from this point onwards, we focus on the trilevel case, avoiding any generalized discussion for M-ary case (setting  $a_1 = 1/2$  and the rest of  $a'_i s = 0$ ). From (2), a trivial bond on the selection of b and g is:

$$0 \le b + \frac{g}{2} \le 1 \tag{4}$$

Constraining the haftoning process such that, no overlapping of the black and gray pixels is allowed, we have:

$$0 \le b, g \le 1 \tag{5}$$

Combined with the additional constraints, in (5), we have a tighter bound, given by:

$$0 \le b + g \le 1 \tag{6}$$

Instead of going into the analytical details of the transform, we resort to simulations, based on graphical interpretations, using the bound defined in (6). Fig. 2(a) and (b) show two new distribution curves, out of the many possible distributions.

In the new multitoning scheme, we first apply the gray level transform to decompose the input image into the black and gray channels, using any conceivable ink distribution, that is within the bounds specified in (6). This gray level separation allows us to visualize the input image as a composition of multiple channels (2 channels in



*Figure 3: New Multitoning Algorithm based on gray level separation.* 

case of trilevel halftoning where only black and gray inks are available).

The gray and black channels are halftoned using conventional Floyd Steinberg error diffusion, with the constraint that, none of the pixels of the two channels overlap each other, thus, preserving the mean gray level. Overlapping of the pixels result in a shift in the gray level, requiring gray level correction as a post processing step, which is an overhead.

In order to avoid the overlapping of black and gray pixels in the output (halftoned) image, the thresholding stages for each channel are conditioned such that, the channel with highest accumulated error value  $x_{a,i}[m, n]$ , will contribute a pixel in the final halftoned image, provided the value is greater than the threshold. The decision rule for the quantizer can be, mathematically, stated as follows:

$$y_{i}[m,n] = \begin{cases} 1 & x_{a,i}[m,n] > x_{a,j}[m,n] & \text{and} \\ & x_{a,i}[m,n] > 0, & i \neq j \\ 0 & \text{otherwise} \end{cases}$$
(7)

No overlapping, of the gray and black channel pixels, is ensured by conditioning the quantizer operation in accordance with (7). The block diagram for the new multitoning scheme is shown in Fig. 3.

Finally, the independently halftoned channels have to be recombined to give the final halftoned image. Let  $b_h$  and  $g_h$  be the two independently halftoned channels that have to be recombined to form the final halftoned output. The recombination equation for trilevel halftoning is as follows:

$$y = b_h + \frac{g_h}{2} \tag{8}$$

# 3. Experimental Results and Discussion

All experiments have been performed on gray scale ramp, using the trilevel halftoning scheme, based on gray level separation. Error filter used is the one with conventional



Figure 4: Gray Scale Ramps halftoned using the new multitoning technique with different conceivable gray level distributions.

Floyd Steinberg weights, with 50% noise added to the weights to account for the texture at gray levels 0.25 and 0.75, due to patterned arrangement of dots near these gray levels. Floyd Steinberg weights, inherently, suffer from this patterning problem at intermediate gray levels and this texture should not be confused with the banding effect in multilevel halftoning.

Fig. 4 shows the gray scale ramps, halftoned using the new multitoning technique. The corresponding gray level distributions are shown in Fig. 5.

The result in Fig. 4(a) is similar to the result generated using the conventional multilevel error diffusion algorithm, the only difference being that in case of the conventional multitoning scheme the quantizer is responsible for the dot growth pattern and in our scheme the dot growth pattern is determined using the gray level distributions in the separator (decomposition unit). Fig. 5(b) shows the corresponding ink distribution curves.

Fig. 4(b) shows the result for halftoning using the proposed scheme, using a different gray level distribution. As shown in Fig. 5(b), the peak of the 'g' curve is flattened in this case, thus, demanding the introduction of black pixels in this flat region to maintain the gray level balance. This



Figure 5: Gray Level distributions corresponding to the gray scale ramps shown in Fig. 4.

result in the new distribution of the 'b' curve, as shown. This redistribution result in the introduction of black and white pixels in the banding region. In the gray scale ramp the black pixels extend to the right of the gray level 0.5 and vice versa for the white pixels, thus, eliminating the band.

In Figs. 5(c) and (d) the peak of the 'g' curve has been further flattened to allow more penetration of the black and white pixels to either side of gray level 0.5. A limiting case would give a result, similar to the bilevel case, where the black and the white pixels extend to the ends, thus, eliminating the use of gray ink.

## 4. Conclusion

In this paper, we have introduced a novel multilevel halftoning technique, based on gray level separation, with the goal of eliminating the unwanted banding artifacts near the intermediate gray levels in the halftoned image. To achieve the gray level separation, we have introduced a gray level transform. Experiments have been performed on gray scale ramps and a comparative discussion has been presented, with the conventional trilevel error diffusion. The results clearly indicate that, using the new multitoning scheme, any conceivable dot growth pattern can be obtained, depending upon the gray level transform, used in the decomposition unit.

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# 6. Bibliography

Faraz Faheem joined the University of Delaware, as a Graduate Student, in the Fall of 1999, after completing his B.E. (Electrical Engineering) from NED University of Engineering and Technology, Karachi, Pakistan, with highest distinction, in December 1997. Faraz has worked with the Digital Telephony Division of Systek (Pvt) Ltd, Karachi, Pakistan, as a Product Design and Developement Engineer and has also spent a year as Visiting Lecturer at NED University. He is currently working in the Lexmark Advanced Digital Display Laboratory, at the University of Delaware, as a Research Assistant, with Gonzalo R. Arce and Daniel L. Lau. His research interests include Digital Halftoning, Baseband Communication Systems and Applied Digital Signal Processing.